Macroeconomics III - Recap Lecture

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General info

- Warm up
- Applications: mainly past exams and assignments
- Q&A: take the chance, as everybody can benefit from your questions

True/False Questions

Evaluate whether the following statements are true or false. Explain your answers.

a Ricardian equivalence ensures that, within an overlapping generation framework, using debt or taxes to finance public spending is equivalent, from households' perspective. **False**. *Ricardian equivalence does not hold within OLG frameworks, given that households have a fixed time horizon.*

True/False Questions

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a Ricardian equivalence ensures that, within an overlapping generation framework, using debt or taxes to finance public spending is equivalent, from households' perspective. **False**. *Ricardian equivalence does not hold within OLG frameworks, given that households have a fixed time horizon.*

b Irrespective of whether it pursues a commitment or a discretionary policy, a central bank can offset demand shocks, while trading-off supply shocks depending on the relative weight of its stabilization objectives in the policy function. **True.** *In fact, demand shocks do not affect the solution of both inflation and output in either regime, while supply shocks increasingly (decreasingly) affect inflation (output) as the weight attached to output fluctuations in the central bank's loss function increases (in either regime).*

True/False Questions

Evaluate whether the following statements are true or false. Explain your answers.

a Ricardian equivalence ensures that, within an overlapping generation framework, using debt or taxes to finance public spending is equivalent, from households' perspective. **False**. *Ricardian equivalence does not hold within OLG frameworks, given that households have a fixed time horizon.*

b Irrespective of whether it pursues a commitment or a discretionary policy, a central bank can offset demand shocks, while trading-off supply shocks depending on the relative weight of its stabilization objectives in the policy function. **True.** *In fact, demand shocks do not affect the solution of both inflation and output in either regime, while supply shocks increasingly (decreasingly) affect inflation (output) as the weight attached to output fluctuations in the central bank's loss function increases (in either regime).*

c The higher menu costs, the lower the degree of price rigidity. **False.** *The higher menu costs, the easier for firms to keep their prices unchanged after a (reasonably sized) shock occurs.*

True/False Questions

d In a model with predetermined prices, each price setter sets the same price for each period over the time span in which she will not be able to adjust prices. **False.** *In a model with predetermined prices, such as the Fisher model, each price setter is allowed to set different prices for each period until a new reset opportunity materializes.*

True/False Questions

d In a model with predetermined prices, each price setter sets the same price for each period over the time span in which she will not be able to adjust prices. **False.** *In a model with predetermined prices, such as the Fisher model, each price setter is allowed to set different prices for each period until a new reset opportunity materializes.*

e In a repeated interaction between the central bank and the rest of the economy over an infinite time horizon, the potential for the benefits from deviating from the commitment policy to be outweighed by the subsequent costs increases in the discount factor. **True.** *Actually, this is true even in a finite time-horizon setting, the reason being that the higher the degree of patience, the higher the weight attached to the costs unraveling over time after the central bank deviates from the announced path.*

True/False Questions

d In a model with predetermined prices, each price setter sets the same price for each period over the time span in which she will not be able to adjust prices. **False.** *In a model with predetermined prices, such as the Fisher model, each price setter is allowed to set different prices for each period until a new reset opportunity materializes.*

e In a repeated interaction between the central bank and the rest of the economy over an infinite time horizon, the potential for the benefits from deviating from the commitment policy to be outweighed by the subsequent costs increases in the discount factor. **True.** *Actually, this is true even in a finite time-horizon setting, the reason being that the higher the degree of patience, the higher the weight attached to the costs unraveling over time after the central bank deviates from the announced path.*

f In a fully funded social security system, the young cohort of households pays social contributions that serve to cover the benefits of the co-existing old generation. **False.** *That defines the pay as you go system, while in a fully-funded system each young cohort provides for itself when old.*

OLG models

Background:

- Examining consumption dynamics and capital accumulation by keeping intergenerational issues in the background
- Transmission of fiscal shocks
- Social security systems
- Ricardian equivalence

January 2017 Exercise 1

Question a

Find the first order conditions for firms' maximization problem that characterize how much capital and labor a firm demands at given factor prices.

January 2017 Exercise 1

Question a

- FOCs of the profit function in per capita terms are

$$
\begin{aligned}
r_t &= f^{'}(k_t) = \alpha A k_t^{\alpha-1} \\
w_t &= f(k_t) - f^{'}(k_t)\, k_t = (1-\alpha) A k_t^{\alpha}
\end{aligned}
$$

January 2017 Exercise 1

Question b

Set up and solve the individual's problem of optimal intertemporal allocation of resources. Derive the Euler equation. Show that individual savings behavior is characterized by

$$s_t = \frac{1-\tau}{2+\rho} w_t - \tau \frac{1+\rho}{2+\rho} \frac{w_{t+1}}{r_{t+1}}.$$

January 2017 Exercise 1

Question b

- The savings problem of a young individual is (recall that $R = 1 + f^{'}(k) - \underbrace{\delta}_{=1}$)

$$\max_{c_{1t}, c_{2t+1}} \ln c_{1t} + \frac{1}{1+\rho} \ln c_{2t+1}$$
$$c_{1t} + s_t = (1-\tau) w_t$$
$$c_{2t+1} = s_t r_{t+1} + \tau w_{t+1}$$

- Solving this problem and combining FOCs yields the Euler equation

$$c_{2t+1} = \frac{r_{t+1}}{1+\rho} c_{1t}$$

- Replace c_{1t} and c_{2t+1} from the budget constraints to obtain the desired equation describing individual savings behavior:

$$s_t = \frac{1-\tau}{2+\rho} w_t - \tau \frac{1+\rho}{2+\rho} \frac{w_{t+1}}{r_{t+1}}.$$

January 2017 Exercise 1

Question c

Show that the capital accumulation equation that expresses k_{t+1}, as a function of k_t, is given by

$$k_{t+1} = \frac{1}{1 + \frac{1+\rho}{2+\rho}\frac{(1-\alpha)}{\alpha}\tau}\frac{(1-\alpha)(1-\tau)}{2+\rho}Ak_t^{\alpha}.$$

January 2017 Exercise 1

Question c

- To derive the capital accumulation equation we use individual savings and replace $k_{t+1} = s_t$ (there is no population growth term here since by assumption $n = 0$), and use the equilibrium expressions for wages and rental rates to obtain

$$k_{t+1} = \frac{1-\tau}{2+\rho}(1-\alpha)Ak_t^{\alpha} - \tau\frac{1+\rho}{2+\rho}\frac{1-\alpha}{\alpha}k_{t+1}.$$

- Combine terms with k_{t+1} to get the desired expression

January 2017 Exercise 1

Question d

Find the level of capital in the steady state.

January 2017 Exercise 1

Question d

Imposing the steady state we get

$$\bar{k} = \left[\frac{1}{1 + \frac{1+\rho}{2+\rho} \frac{1-\alpha}{\alpha} \tau} \frac{(1-\alpha)(1-\tau)}{2+\rho} A \right]^{\frac{1}{1-\alpha}}$$

January 2017 Exercise 1

Question e

Assume that the economy is initially in the steady state. Unexpectedly, at time $t = T$, the government decides to dismantle the social security system from period $T + 1$ onwards: No contributions will be raised and no benefits will be paid, at any point in the future. Importantly, the policy shift is communicated to the public after savings decisions have been formulated. What is the expression for the new steady state capital level? Will the stock of capital be higher at $T + 2$, as compared with $T + 1$?

January 2017 Exercise 1

Question e

- From the previous point we can show that, for $\tau = 0$, the new steady state satisfies

$$\bar{k}_{NEW} = \left(\frac{1-\alpha}{2+\rho} A \right)^{\frac{1}{1-\alpha}}$$

- Recall that capital is predetermined. Thus $k_{T+1} = s_T = \bar{s} = \bar{k}$. As for k_{T+2}, the new law of motion reads as:

$$k_{T+2} = \frac{1-\alpha}{2+\rho} A k_{T+1}^{\alpha}$$

which implies that k_{T+2} will be greater than the original steady state, $\bar{k}$, as long as

$$\bar{k} < \left(\frac{1-\alpha}{2+\rho} A \right)^{\frac{1}{1-\alpha}} = \bar{k}_{NEW}$$

- Impact of the deep parameters

January 2017 Exercise 1

Question f

Is the old generation at time $T+1$ *better-off or worse-off, after the social security system has been dismantled?*

January 2017 Exercise 1

Question f

As $\tau = 0$ becomes effective starting from $T+1$ onwards, the old generation in $T+1$ will be worse-off, as these agents have not had the chance to adapt their consumption/saving profile (given that the government has communicated the decision after they had already optimized over their consumption/saving decisions), while the co-existent young generation will not be covering their social benefits.

January 2017 Exercise 1

Question g

Has the economy higher chances of being dynamically inefficient with or without a pay-as-you-go social security system? Explain.

January 2017 Exercise 1

Question g

- Dynamic efficiency obtains under a situation in which the rate of return is greater than the return on social security, which is implicitly measured by population growth (assumed to be zero, in this case)
- Thus, as the rate of return is a negative function of the capital stock ($r_t = \alpha A k_t^{\alpha-1}$) and the latter is negatively affected by τ, we infer that a pay-as-you-go social security system has higher chances of depressing capital accumulation and thus rendering the system dynamically efficient

Real Business Cycles

Background:

- The second part of the course has mainly focused on equilibrium theories of business fluctuations
- In this respect, the RBC model still represents a key tool for examining comovements and persistence of main macroeconomic quantities
- Unifying theory of growth and fluctuations (recall Solow's neoclassical growth model as the key reference)
- Focus on the propagation mechanisms (especially over time) of shocks relies on intertemporal substitution effects
- Technological shocks as a main driving force of business cycle fluctuations

June 2015 Exercise 2

Question a

Set up the representative firm's and household's optimization problems and derive the necessary first order conditions, respectively.

June 2015 Exercise 2

Question a

- We first set up the Lagrangian for households' optimization:

$$\mathcal{L}_t = \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log c_t - \frac{h_t^{1+\nu}}{1+\nu} + \lambda_t \left[w_t h_t + \mu_{t-1}(d_t + q_t) - c_t - q_t \mu_t \right] \right\} \tag{1}$$

  The first order conditions with respect to the choice variables (c_t, h_t, μ_t) are:

$$\frac{\partial \mathcal{L}_t}{\partial c_t} = 0 \Longrightarrow \frac{1}{c_t} - \lambda_t = 0 \tag{2}$$

$$\frac{\partial \mathcal{L}_t}{\partial h_t} = 0 \Longrightarrow -h_t^{\nu} + \lambda_t w_t = 0 \tag{3}$$

$$\frac{\partial \mathcal{L}_t}{\partial \mu_t} = 0 \Longrightarrow -q_t \lambda_t + \beta \mathbf{E}_t \left[\lambda_{t+1}(d_{t+1} + q_{t+1}) \right] = 0 \tag{4}$$

- Firms' objective is to maximize lifetime discounted profits. However, since there are no dynamic links across periods, this is the same as maximizing profits in every periods. Specifically, the representative firm chooses labor hours so that

$$\max_{h_t} \left(z_t h_t^a - w_t h_t \right) \tag{5}$$

  which leads to the necessary condition:

$$\alpha z_t h_t^{a-1} = w_t \tag{6}$$

June 2015 Exercise 2

Question b

Characterize the labor demand and supply schedules and prove that the equilibrium wage and hours are $w_t = \alpha^{\frac{a+v}{1+v}} z_t$ *and* $h_t = \alpha^{\frac{1}{1+v}}$*, respectively.*

June 2015 Exercise 2

Question b

- To characterize the labor market equilibrium, we take (6) as the labor demand function. As to the supply side of the labor market, combine (2) and (3) so as to get:

$$h_t^{\nu} = \frac{w_t}{c_t} \tag{7}$$

- We then employ the market clearing condition $y_t = c_t$ and the technology constraint $y = z_t h_t^{\alpha}$ to substitute for h_t and c_t in (7):

$$h_t = \left(\frac{w_t}{z_t}\right)^{\frac{1}{\nu+\alpha}} \tag{8}$$

- To find equilibrium wage and hours, we equalize (6) and (8), obtaining:

$$h_t = \alpha^{\frac{1}{1+\nu}} \tag{9}$$

$$w_t = \alpha^{\frac{a+\nu}{1+\nu}} z_t \tag{10}$$

June 2015 Exercise 2

Question c

Find the equilibrium value of dividends. Following a positive realization of the technology shock (i.e., $z_t > 0$), equilibrium dividends are negative. True or false? Why?

June 2015 Exercise 2

Question c

- We replace the equilibrium levels of h_t and w_t so as to get dividends in equilibrium:

$$d_t = z_t \alpha^{\frac{\alpha}{1+v}} \left(1 - \alpha^{1+v}\right) \tag{11}$$

- The statement is false given that, at best, $\alpha = 1$, in which case profits are null

June 2015 Exercise 2

Question d

Starting from the first order condition with respect to μ_t, prove that

$$q_t = \beta \mathbf{E}_t \left[\frac{z_t \alpha^{\frac{\alpha}{1+v}}}{z_{t+1} \alpha^{\frac{\alpha}{1+v}}} \left(z_t \alpha^{\frac{\alpha}{1+v}} \left(1 - \alpha^{1+v}\right) + q_{t+1} \right) \right] \tag{12}$$

Under which value for α equation (12) reduces to $\frac{q_t}{z_t} = \beta \mathbf{E}_t \left[\frac{q_{t+1}}{z_{t+1}} \right]$?

June 2015 Exercise 2

Question d

- Combine (4) with (2), so as to get the following equilibrium condition for the stock price:

$$q_t = \beta \mathbf{E}_t \left[\frac{c_t}{c_{t+1}} \left(d_{t+1} + q_{t+1} \right) \right] \tag{13}$$

- In the latter, we replace c_t, c_{t+1} and d_{t+1} with their equilibrium values, obtaining

$$q_t = \beta \mathbf{E}_t \left[\frac{z_t \alpha^{\frac{\alpha}{1+v}}}{z_{t+1} \alpha^{\frac{\alpha}{1+v}}} \left(z_t \alpha^{\frac{\alpha}{1+v}} \left(1 - \alpha^{1+v} \right) + q_{t+1} \right) \right] \tag{14}$$

- It is immediate to verify that (14) reduces to $\frac{q_t}{z_t} = \beta \mathbf{E}_t \left[\frac{q_{t+1}}{z_{t+1}} \right]$ for $\alpha = 1$, i.e. when dividends are zero

Nominal Rigidities

Background:

- Background: Lucas (1972) model of imperfect information
- Microfoundation of a relationship between inflation and output (Phillips curve)
- Expected changes in aggregate demand only affect prices, while unexpected changes affect prices and output, i.e. they have real effects

Policy implications:

- Monetary policy can stabilize real activity only if policy-makers have information that is not available to private agents
- *Lucas critique* (1976): we cannot mechanically transpose past behavior into the future

Empirical predictions:

- The Lucas (1972) model predicts that in economies with high aggregate demand volatility the real effects of a given change in aggregate demand should be smaller
- Lucas (1973) tests this prediction using cross-country data
- Although there is some positive evidence, later studies show that nominal rigidities in price setting have more explanatory power
- Move away from competitive behavior and assume firms have *market power in setting prices*

Nominal Rigidities

Models:

- Fisher (baseline: each price-setter sets prices for two periods, rational expectations in price setting)
- Taylor: Fisher + assumption that a firm setting prices at time t for periods t and $t+1$ is forced to choose same prices for both periods
- Calvo: Taylor model + assumption that every period firms are able to set new prices, but only with probability $0 < \alpha \leq 1$
- Menu costs: fixed cost of changing prices (evaluation of costs vs benefits from changing prices)

February 2017 Exercise 2

Question 1

Derive the equilibrium level of (log) output y and show that the equilibrium (log) aggregate price level equals

$$p = \mu + m$$

where μ is a constant to be derived. [Hint: assume that each producer charges the same price, and that the price index p equals this common price.]

February 2017 Exercise 2

Question 1

- Substitute the budget constraint, the technology constraint and the demand function into the utility function, so as to get:

$$U_i = \left(\frac{Y_i}{Y}\right)^{-\frac{1}{\eta}} Y_i - \frac{1}{\lambda} Y_i^{\lambda}$$

- Maximizing w.r.t. Y_i:

$$\frac{\partial U_i}{\partial Y_i} = 0 \Rightarrow -\frac{1}{\eta}\left(\frac{1}{Y}\right)^{-\frac{1}{\eta}} (Y_i)^{-\frac{1}{\eta}-1} Y_i + \left(\frac{1}{Y}\right)^{-\frac{1}{\eta}} (Y_i)^{-\frac{1}{\eta}} - Y_i^{\lambda-1} = 0$$

February 2017 Exercise 2

Question 2

- FOC:

$$\left(1 - \frac{1}{\eta}\right) \frac{P_i}{P} = Y_i^{\lambda - 1}$$

- Taking logs and rearranging to bring y_i on the LHS:

$$y_i = \frac{1}{\lambda - 1} (p_i - p) + \frac{1}{\lambda - 1} \ln \left(1 - \frac{1}{\eta}\right) \tag{15}$$

- We know that each producer charges the same price, and that the general price index equals this common price. Thus:

$$y = \frac{1}{\lambda - 1} \ln \left(\frac{\eta - 1}{\eta}\right)$$

- Since $y = m - p$:

$$p = m - y = m - \frac{1}{\lambda - 1} \ln \left(\frac{\eta - 1}{\eta}\right)$$

- Therefore:

$$\mu = -\frac{1}{\lambda - 1} \ln \left(\frac{\eta - 1}{\eta}\right)$$

February 2017 Exercise 2

Question 2

Show that the desired (log) price level for good i equals

$$p_i^* = c + \phi m + (1 - \phi)\, p \tag{16}$$

where ϕ and c are constants to be derived.

February 2017 Exercise 2

Question 2

- As for the desired price at the individual level, we resort to (15)
- Imposing the notation $\phi \equiv \lambda - 1$, we obtain:

$$p_i^* = \phi m + (1 - \phi)\, p + c$$

where $c = \phi\mu$

February 2017 Exercise 2

Question 3

Without loss of generality, set $c = 0$ from now on. Suppose now that the price p_i is fixed for two periods and that price setting is staggered, such that half of prices $p_{i,t}$ are set in period t at level x_t, and the other half were set in period $t-1$ at level x_{t-1}. Thus, the aggregate price level equals

$$p_t = \frac{1}{2}\left(x_t + x_{t-1}\right)$$

Suppose that the (log) money supply is a white noise. Assuming certainty equivalence (i.e., $x_t = \frac{1}{2}\left(p^*_{i,t} + \mathbf{E}_t\left[p^*_{i,t+1}\right]\right)$), show that the following difference equation can be obtained to define the dynamics of x_t:

$$x_t = \frac{2\phi}{1+\phi} m_t + \frac{1-\phi}{2\left(1+\phi\right)}\left(\mathbf{E}_t\left[x_{t+1}\right] + x_{t-1}\right).$$

February 2017 Exercise 2

Question 3

- Set $\phi\mu = 0$. Assuming certainty equivalence:

$$x_t = \frac{1}{2}\left(p^*_{i,t} + \mathbf{E}_t\left[p^*_{i,t+1}\right]\right)$$

- Thus

$$x_t = \frac{1}{2}\left(\phi m_t + (1-\phi)\, p_t + \phi \mathbf{E}_t\left[m_{t+1}\right] + (1-\phi)\,\mathbf{E}_t\left[p_{t+1}\right]\right)$$

- Recall now that $p_t = \frac{1}{2}(x_t + x_{t-1})$ and that m_t is a white noise shock. The equation above becomes

$$x_t = \frac{\phi}{2} m_t + \frac{1}{2}\left((1-\phi)\frac{1}{2}(x_t + x_{t-1}) + \frac{1}{2}(1-\phi)\,\mathbf{E}_t\left[(x_{t+1} + x_t)\right]\right)$$

which translates into

$$x_t = \frac{\phi}{1+\phi} m_t + \frac{1-\phi}{2(1+\phi)}\left(\mathbf{E}_t\left[x_{t+1}\right] + x_{t-1}\right) \qquad (17)$$

February 2017 Exercise 2

Question 4

Guess a solution for x_t of the type $x_t = \beta x_{t-1} + \gamma m_t$, and find the equilibrium values of β and γ, conditional on setting $\lambda = 1$.

February 2017 Exercise 2

Question 4

- Guess a solution for x_t of the type:

$$x_t = \beta x_{t-1} + \gamma m_t$$

- Plug this into (17) to eliminate $\mathbf{E}_t\left[x_{t+1}\right]$:

$$x_t = \frac{2\phi}{2\left(1+\phi\right) - \left(1-\phi\right)\beta} m_t + \frac{1-\phi}{2\left(1+\phi\right) - \left(1-\phi\right)\beta} x_{t-1}$$

from which we can infer

$$\begin{aligned} \gamma &= \frac{2\phi}{2\left(1+\phi\right) - \left(1-\phi\right)\beta} \\ \beta &= \frac{1-\phi}{2\left(1+\phi\right) - \left(1-\phi\right)\beta} \end{aligned}$$

February 2017 Exercise 2

Question 4

- Thus, we need to solve the system above, conditional on $\lambda = 1$, which implies $\phi = 0$:

$$\begin{aligned} \gamma &= 0 \\ \beta &= \frac{1}{2 - \beta} \end{aligned}$$

- so that β solves the quadratic equation

$$\beta^2 - 2\beta + 1 = 0$$

whose unique solution is $\beta = 1$